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1993 J. Phys. A: Math. Gen. 26 6187

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## Life in one dimension: statistics and self-organized criticality

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Received 7 June 1993, in final form 16 August 1993

**Abstract.** The controversy regarding the occurrence of self-organized criticality in the cellular automaton Game of Life has not yet been resolved, mainly due to its massive computational requirements. We consider a one-dimensional version of Life which shows essentially the same local complexity as its two-dimensional counterpart but allows a more extensive computational implementation. General cluster statistics, geometrical properties and self-organized criticality are investigated. Implications concerning higher-dimensional Life are discussed.

The concept of self-organized criticality (soc) [1] has proven extremely fecund in the approach of non-equilibrium extended spatial systems and is finding applications in a wide variety of phenomena very little connected until recently, such as economics and astrophysics. The basic property of soc is that power laws characterize the relaxation events for the system in the critical state. Furthermore, such a condition would result without fine tuning so that the soc state would be the natural attractor to which the system is driven by the dynamics.

A particularly interesting result has been the suggestion [2] that the Game of Life [3, 4] would exhibit soc. Life is a cellular automaton defined on a square lattice of  $L^2$  sites with possible values 0 or 1. In each time step the fate of each site is dependent on its eight nearest neighbours and parallel updatings according to the following rules: (i) a live site (1) will still be alive in the next generation if it has two or three live neighbours, otherwise it will die ( $1 \rightarrow 0$ ); (ii) a vacant site (0) will be occupied in the next generation ( $0 \rightarrow 1$ , birth) if it has exactly three neighbours. This simple set of rules yields amazingly complex structures [3, 4]. The occurrence of soc in Life is interesting because it is a non-conservative model and has an important biological motivation. However, other simulations have been performed [5] which seemed to indicate that Life would in fact not be a soc model. The question has proven controversial [6] and is not yet resolved. A major difficulty in elucidating the problem resides in the highly demanding computational capacity required for the realization of extensive simulations in large lattices which require large stabilization times and in many cases the use of sophisticated machinery [6]. With such points in mind we study here a qualitatively very similar version of Life defined in one dimension [7, 8]. The low dimensionality makes possible implementations on large lattices and hence a more systematic and reliable statistical study of the game.

The one-dimensional Life (to which we will refer as  $1GL$ ) is defined on a line of  $L$  sites with values as in the two-dimensional game ( $2GL$ ). The fate of a given site is defined by its four nearest neighbours, parallel updatings, and the following rules:

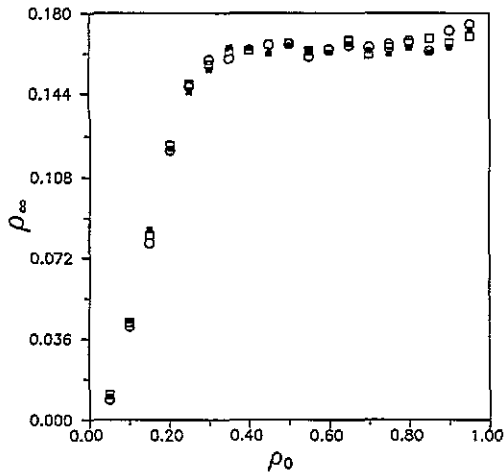


Figure 1. Asymptotic density  $\rho_\infty$  as a function of initial density  $\rho_0$ , for  $0.05 \leq \rho_0 \leq 0.95$  in steps of 0.05 and a total of 100 experiments in each case. Points for  $L = 10^4$  ( $\circ$ ),  $10^5$  ( $\square$ ), and  $10^6$  ( $\blacksquare$ ) are all superimposed. Two regimes are noticed: region I (of dependence on initial occupancy) and region II (plateau).

(i) a live site with either two or four live neighbours will survive in the next generation, otherwise it will die; (ii) a vacant site with either two or three live neighbours will be occupied (birth) in the next generation. With these rules the 1GL shows all essential local behaviour of the 2GL, such as the existence of propagating and self-reproducing structures. Numerous examples of the rich fauna of the 1GL are presented in [7]. Since the 2GL was introduced, virtually all efforts have concentrated on the understanding of its local properties. From this point of view the 1GL presents analogous behaviour and so may be considered as a valid construction but more easily implemented. Only recently global properties of Life have been investigated more extensively [2, 9–11]. Our goal here will be to study numerically the global properties of the 1GL. All simulations were carried on Sun SPARC workstations and the rules of the game were implemented using the simple algorithm presented in [8].

A detailed description of the system is provided by the size distribution function  $n(s, t)$  giving the number of clusters of size  $s$  at time  $t$ . A cluster is a set of live sites connected by a first-nearest-neighbour relation. In what follows we describe results from simulations of the 1GL with open boundary conditions in lattices of the sizes up to  $L = 10^6$  from initial uncorrelated configurations at time  $t = 0$ .

Two important quantities to be calculated are the first two moments of  $n(s, t)$ , namely, the mass density  $\rho(t) = \sum_s s n(s, t) / L$  and the total number of clusters (cluster population)  $N(t) = \sum_s n(s, t)$ . As the rules are applied, the 1GL evolves until a stable configuration is attained. Stable means that usual macroscopic variables (mass density, cluster population, average number of neighbours per site) reach a condition of stationary, oscillatory behaviour, or chaotic wandering around an average value. Of great interest to us are the properties of this asymptotic state. We show in figure 1 the dependence of  $\rho_\infty = \rho(t \rightarrow \infty)$  with  $\rho_0 = \rho(t = 0)$ . Two distinct regimes are readily observed: for  $0 \leq \rho_0 < 0.30$  (region I) the mass density in the stabilized regime grows rapidly while for  $0.30 \leq \rho_0 < 1.00$  (region II)  $\rho_\infty$  remains a constant (within statistical uncertainties) around 0.162. Such behaviour is quite similar to that found in the 2GL case [9]. An essential difference is found, however, for high initial occupancies where

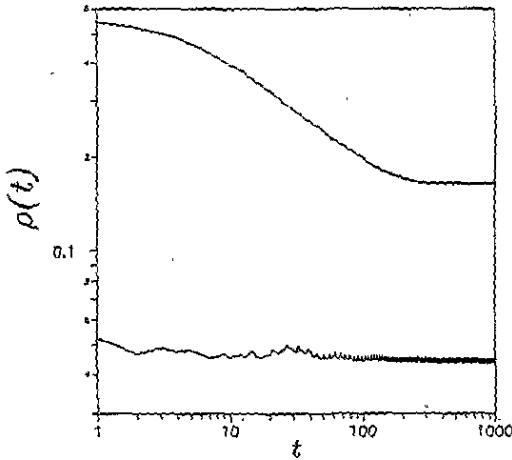


Figure 2. Typical population density  $\rho(t)$  as a function of time  $t$  for  $\rho_0$  in region I (lower curve) and region II (upper curve).

$\rho_\infty$  does not fall to zero. This may be understood if we recall that for the 1GL a completely populated universe is a stable state of the dynamics. From figure 1 it can be seen that  $\rho_\infty$  is independent of  $L$ . On the other hand  $N_\infty = N(t \rightarrow \infty)$  scales with lattice size as  $N_\infty \sim L^{1.00 \pm 0.06}$  but has a similar functional dependence on  $\rho_0$  as  $\rho_\infty$ . The dynamical evolution of the density is shown in figure 2 for values of initial density from regions I and II. In region II the mass density decays following a power law given by  $\rho(t) \sim t^{-0.5 \pm 0.01}$  after an initial transient and just before the asymptotic state is reached. The transient interval consists of about the first 10 time steps where the strong correlations with the initial configuration are destroyed. The same is verified for the cluster population where  $N(t) \sim t^{-0.19 \pm 0.03}$ . In region I the system is so sparsely distributed that competition between clusters turns out to be an effect of minor importance. As a consequence no interesting time dependence for  $\rho$  or  $N$  is found.

A notorious property of Life is its capacity to generate complexity. A particularly useful measure of complexity suitable for lattice problems is the diversity of clusters originally introduced in fragmentation dynamics [12–14] and more recently in Life [11]. The diversity  $\Delta(t) = \sum_s \theta\{n(s, t)\}$ , with  $\theta(x) = 1$  if  $x > 0$  and 0 otherwise, gives the number of different clusters at time  $t$ . The evolution of diversity is shown in figure 3. Over an interval approximately three times larger than that of power law dependence for  $\rho$  the diversity shows an exponential decay with  $\Delta(t) \sim e^{(-0.003 \pm 0.0005)t}$  in region II. We have found that  $\Delta_\infty$  is independent of both  $L$  and  $\rho_0$  with an average value  $\Delta_\infty = 3.2 \pm 0.7$ . This implies that a relatively small diversity is found in the stable state coming from a collection of small animals distributed over the line. In fact usually only objects of size no greater than about 5 populate the asymptotic states. From these facts it is possible to calculate directly the mass-size exponent  $D$  of the distribution of live sites in the asymptotic state. The relation  $M_\infty \sim L^D$ , with  $M_\infty$  the total mass at  $t \rightarrow \infty$ , defines the exponent  $D$ . Since  $M_\infty = \sum_s sn(s, t_\infty) \cong \sum_s n(s, t_\infty) = N_\infty \sim L^1$  we find  $D = 1$ . Alternatively,  $\rho_\infty \sim L^{D-1}$  and  $\rho_\infty$  is independent of  $L$ , implying that  $D = 1$ . The measurement of the radius of gyration  $R_g = [\sum_{j>i} |r_i - r_j|]^2 / M_\infty$ , with  $r_i$  the position of a live site  $i$ , can make these results more rigorous. From simulations we have determined that  $R_g = L/3.5$  and  $M_\infty \sim R_g^{1.04 \pm 0.06}$  and so the 1GL distributes on a disconnected set of dimension 1.

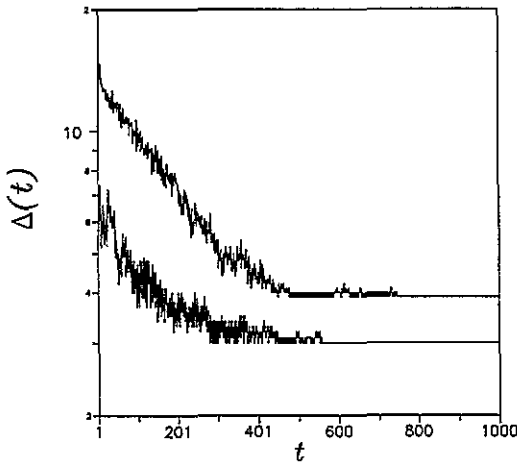
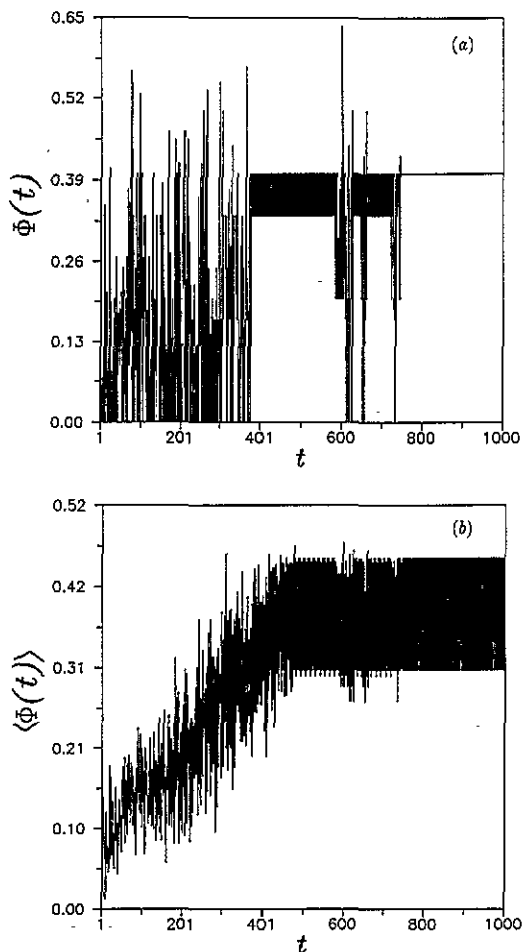


Figure 3. Evolution of diversity of cluster sizes for region I (lower curve) and region II (upper curve).

Another quantity that may shed light on how the system organizes over the possible states of cluster size is the ‘filling’ function  $\Phi(t) = [s_{\max}(t) - \Delta(t)]/s_{\max}(t)$ , where  $s_{\max}(t)$  is the biggest cluster size found at time  $t$ . The filling function expresses dynamically the evolution of lacunarity in size occupation. This is shown in figure 4 for a single experiment (a) and averaged over many cases (b). A tendency of linear growth with time is observed for the average  $\langle \Phi(t) \rangle$  on the same interval of exponential decay for the diversity. We have noticed that even when the system has evolved to a configuration where practically all macroscopic quantities are relatively stabilized near their asymptotic values, the filling function may present occasional ‘bursts’ of apparently chaotic activity exhibiting behaviour reminiscent of intermittency. This is a result of the strong sensitivity of the filling function to localized interactions.

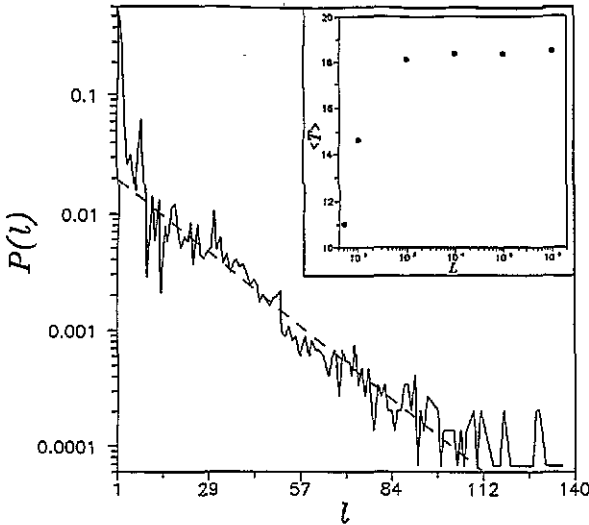
To study the possibility of SOC in the 1GL case we follow the usual procedure of perturbing the stable configurations and determining the resulting distribution of relaxation times and avalanche sizes. In our case the perturbation is the addition of a live site at a randomly chosen vacant site. As a result, the system is driven away from the stable state and will tend to reorganize in another stable configuration after  $T$  updatings (the stabilization time). Until a new stable state is achieved the perturbation affects the fate of its neighbours propagating its influence over a length  $l$  (the avalanche size). This quantity gives a measure of how correlated are the structures of the system: in the SOC state, where presumably we have global connectivity, we should expect single perturbations to cause events in all scales from very localized up to very large ‘catastrophic’ ones, according to a power law distribution characterized by a robust exponent. The same behaviour is expected for the distribution of stabilization times. In describing our results we need to consider two factors: lattice size and initial occupancy. In region I the added perturbation site almost always disappears in the first applications of the updating rules and the stabilization time is very small, independent of lattice size. Although for region II non-trivial effects can happen, these are far from what is expected for a SOC state. Large-scale events spanning all the system are never present and in fact no avalanche length greater than 500 has been detected independent of lattice size, as long as  $L > 10^3$ . Also the average stabilization



**Figure 4.** (a) Typical behaviour of the filling function (see text)  $\Phi(t)$  in a single experiment. (b) Averaged filling function  $\langle\Phi(t)\rangle$  over an ensemble of 50 equivalent experiments on a lattice of  $10^5$  sites.

time  $\langle T \rangle$  does not increase with lattice size as we should expect [5]. Only for  $L$  of the order of  $10^2$  rare catastrophic avalanches are observed. In figure 5 we show the distribution  $P(l)$  of avalanche sizes and the average stabilization time as a function of  $L$ . Following each perturbation a value for  $l$  and  $T$  is measured. After a large number of experiments are performed we can obtain the probability, expressed by  $P(l)$ , that an avalanche of a given size will occur. The curve shown for  $P(l)$  is typical for  $L > 10^3$  and refers to  $10^5$  experiments in a lattice with  $L = 10^5$ . The exponentially decreasing best fitting curve as well as the dependence of  $\langle T \rangle$  on  $L$  are in conformity with that found for the 2GL case [5] indicating the absence of soc behaviour. The referred size-independent cutoffs in time and space verified over several decades in  $L$  practically rule out the possibility of soc in the 1GL case.

In conclusion, we have studied a one-dimensional version of Life (1GL) with similar local properties as the two-dimensional game (2GL). We have analysed the behaviour of the asymptotic ( $t \rightarrow \infty$ ) states as well as the evolution of mass density, cluster population, diversity, and cluster size occupation. An important motivation was to



**Figure 5.** Distribution  $P(l)$  of avalanche sizes  $l$  triggered by single perturbations. The curve refers to  $10^5$  experiments on a lattice of  $10^5$  available sites but a similar behaviour is observed for  $L > 10^3$ . The insert shows the average stabilization time  $\langle T \rangle$  following a perturbation as a function of  $L$ .

consider the occurrence of self-organized criticality as suggested for 2GL. We have found that in one dimension Life does not seem to exhibit soc behaviour. Evidently we cannot infer from this that the 2GL is not a soc model as well. Nevertheless, it is worth recalling that, in many cases, one-dimensional models may be more complex than higher-dimensional ones [15]. If this also happens in Life it is something that cannot be straightforwardly decided because generalizations of Life always hide a flavour of arbitrariness. However, if one believes (as our results seem to indicate) that the version considered here is valid so that it can reproduce generally the same complexity as the 2GL case, both locally and globally, then the evidence is that Life is not critical in the soc sense, at least for  $d=1$  and  $d=2$ . This fact by no means minimizes the importance of the automaton, which remains to be fully understood and constitutes a most notable source of complexity. Usual features of the game have also motivated higher-dimensional versions [16] that may bring further new possibilities.

### Acknowledgment

The author gratefully acknowledges the Brazilian agency CAPES for financial support and Dr M A F Gomes for comments on initial versions of the manuscript.

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